Empowering mathematics learners

by Ariyadi Wijaya

Chapter XX

Empowering mathematics learners through exploratory tasks

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Mathematical empowerment is often connected to the ability to fluently use mathematical knowledge and skills in various contexts or situations. Consequently, mathematics learning should support students' acquisition of not only concepts, but also skills and various strategies of problem solving. One of the possible ways to empower mathematics learners is through the use of exploratory mathematics tasks. An exploratory mathematics task could lead students to exploratory activity, which includes interpreting situations in mathematical terms, formulating mathematics questions, making conjectures, exploring various strategies, and making generalizations. This chapter provides examples of the use of exploratory mathematics tasks in classroom practices. The tasks did not provide any indication about the required mathematic concepts. In their early work, many students used non-mathematical strategies. The students started exploring mathematics concepts after the teacher scaffold them to use some mathematics to solve the tasks. This finding suggests that exploratory task is not a standalone feature to empower mathematics learners. Teachers' scaffolding is an important requirement to guide students in dealing with exploratory tasks and in developing their ability to use mathematics.

1 Introduction

Empowering mathematics learners has long been considered as an essential goal of mathematics education. In the 1990s, Skovsmose (1994) promoted a so called 'critical mathematics education' in which he highlighted that teachers should implement mathematics curriculum as an empowering activity for their students. A call for mathematical empowerment also approached the policy level. Clements and Ellerton (1996) suggested that reforming mathematics education required a collaborative work between curriculum developers and teachers to develop curriculum that is empowering both for teachers and their students. In recent years, empowering mathematics learners is often connected to mathematical literacy and considered as an important key not only for higher education, but also for economic purposes and full citizenship (Doyle, 2007; Stinson, 2004). This view emphasizes that mathematics should not be seen as an isolated concept that is apart from students' life. Instead, mathematics should be seen and treated as an integral part of students' life.

According to Ernest (2002), mathematical empowerment concerns knowledge and skill to use and apply mathematics. It deals with students' acquisition of mathematical knowledge and skills, and also problem solving skills. Mathematically empowered students could "demonstrate an appropriate range of mathematical capabilities such as performing algorithms and procedures, computing solutions to exercises, solving problems" (Ernest, 2002, p. 2). Furthermore, these students also have a good ability in applying mathematics concepts, carrying out approaches in solving mathematical problems, and judging the correctness of solutions. Doyle (2007) highlighted that a key to mathematical empowerment is mathematical literacy for which mathematics tasks play a crucial role. It is in agreement with Stein, Smith, Henningsen & Silver (2009) who emphasized that cognitive demands of mathematics tasks determine the development of students' mathematical thinking and skills. This chapter discusses the use of exploratory tasks to empower mathematics learners by considering the role of mathematics tasks for mathematical empowerment.

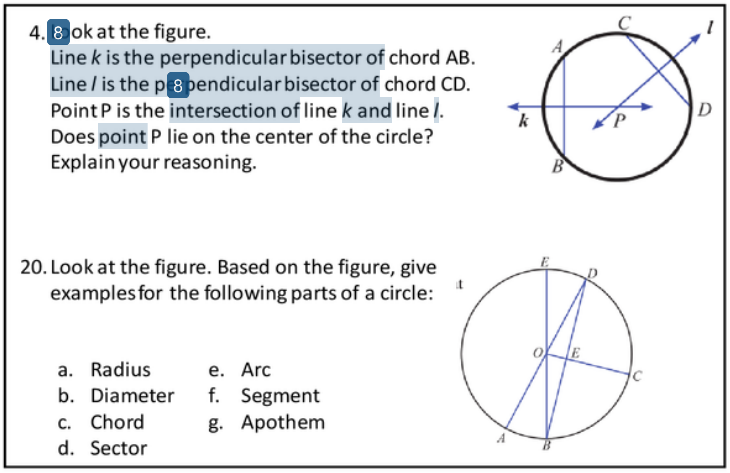
2 Exploratory Tasks

Exploratory tasks are tasks that "may lead students to exploratory activity, from which they do substantial work and learn new mathematics" (Ponte, Mata-Pereira, Henriques, & Quaresma, 2013, p. 11). Exploratory tasks are not only used to support the development of new concepts, but also to develop students' ability to apply mathematics concepts they have learnt and to connect between concepts. Exploratory tasks are ill-structured and do not have straightforward solving strategies (Cifarelli & Cai, 2005). Another characteristic of exploratory tasks deals with the type of information available in the tasks. Exploratory tasks mostly have either superfluous information or incomplete information. Tasks with superfluous information mean that the tasks contain irrelevant data so that the solvers need to identify and select only the relevant data. Tasks with incomplete information indicate that some important data are missing in the task so that the solvers need to gather the required data either by estimation, through multistep procedures, or search from other sources.

To solve exploratory task students need to reformulate the problem statements, interpret the problem situations in terms of mathematics, and generate mathematical model. During their exploratory activity, students are actively involved in exploring strategies or relationships, mathematical reasoning, making conjectures, and communicating mathematical ideas, and interpreting and validating the reasonableness of mathematical results.

3.1 *Designing exploratory tasks*

Most tasks in Indonesian textbooks have low cognitive demands and focus on procedural knowledge (Wijaya, Van den Heuvel-Panhuizen, & Doorman, 2015). Figure 1 shows examples of tasks available in Indonesian mathematics textbooks. Task 20 mainly focuses on factual knowledge, i.e. about the definition of parts of a circle. This task does not offer enough opportunities for students to do exploratory activity. Task 4 requires students to prove a particular mathematical situation. Although this task has high cognitive demands, the exploratory activity is limited to mathematical exploration because the task is situated in intra-mathematical context. In this case, students do not get opportunity to do a wide range of exploratory activity which includes mathematical modeling.


4. Look at the figure.
Line *k* is the perpendicular bisector of chord AB.
Line *l* is the perpendicular bisector of chord CD.
Point P is the intersection of line *k* and line *l*.
Does point P lie on the center of the circle?
Explain your reasoning.

20. Look at the figure. Based on the figure, give examples for the following parts of a circle:

a. Radius
b. Diameter
c. Chord
d. Sector
e. Arc
f. Segment
g. Apothem

Figure 1. Examples of textbook tasks
(Note: translation of Problem 4 and Problem 20, As'ari et al., 2014, p. 81)

By referring to the characteristics of exploratory tasks as mentioned earlier, a regular textbook task as shown in Figure 1 can be modified into an exploratory task. Figure 2 shows an example of exploratory task. To facilitate students' modeling process, this exploratory task is situated in an extra-mathematical context and do not have straightforward strategies. In the case of Figure 2, the context is about an archaeological artifact for which no clear indication about mathematical procedure is provided. Students need to transform this real-world problem into a mathematical problem. At this stage, students' exploratory activity focuses on interpreting the mathematical meaning of "reconstructing a broken plate into original size and form". Exploratory activity continues when students already work within mathematical problem or model; for example exploring mathematical concepts or procedures which are relevant to constructing a circle from a given arc. An exploratory task could also lead to hands-on activity or paper-and-pencil exploratory activity.

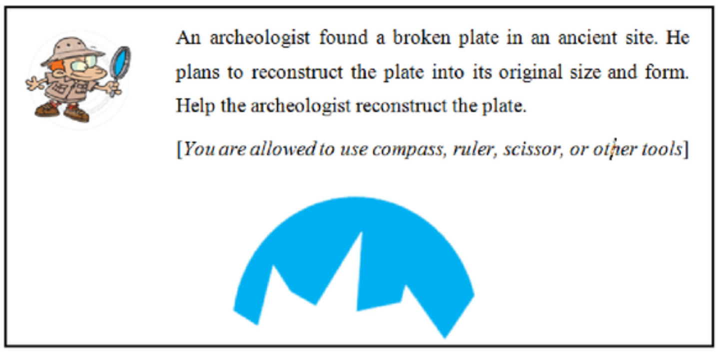


Figure 2. Broken Plate: An example of exploratory task
(adopted from Koremsky (1992) with some adjustments)

3.2 *Computer-based exploratory tasks*

Computer is beneficial for exploratory activity because it, according to Clements (2000), offers student flexibility in exploring various possible strategies and solutions for mathematics problems. Considering this potential of computer, an exploratory task can also be developed in the form of a computer-based task. One of computer program or software that can be used to make a computer-based exploratory task is Geogebra. According to Dikovic (2009), GeoGebra is beneficial for mathematics learning because it: (1) has easy-to-use interface, (2) could encourage students to do mathematical projects, experimental and guided discovery learning, (3) stimulates students to personalize their creations through exploration of features, (4) helps students gain a better understanding of mathematics concepts.

Figure 3 is the Broken Plate Task which is presented as a computer-exploratory task by using GeoGebra. Before students start executing exploratory activity with GeoGebra, they still need to interpret the broken plate task into a mathematical problem. After the students know the mathematical problem of the task they can use the features of GeoGebra to explore strategies for solving the mathematical problem.

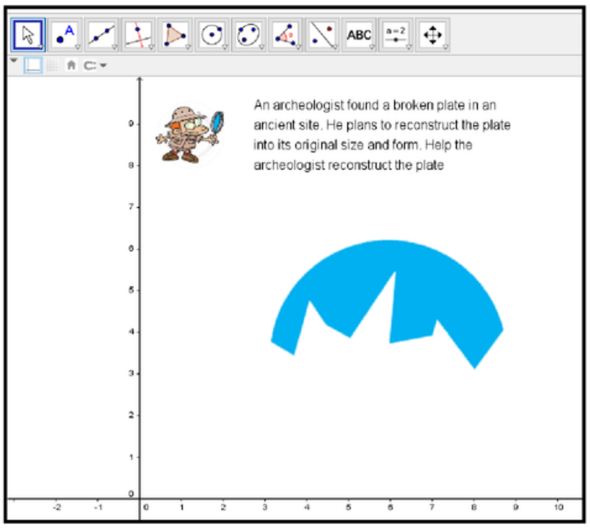


Figure 3. Broken Plate: A GeoGebra-based exploratory task

3 Using Exploratory Tasks in Mathematics Classrooms

This section illustrates the use of exploratory tasks in classrooms. The two forms of Broken Plate Task were used in two different groups of first-year university students who were enrolled in a teacher training program. One group worked with the paper-and-pencil exploratory task and the other group worked with GeoGebra-based exploratory task. Although the participants of this study were university students, the prior geometrical knowledge of these student teachers was relatively similar to that of senior high school students because these student teachers have not yet learned college geometry.

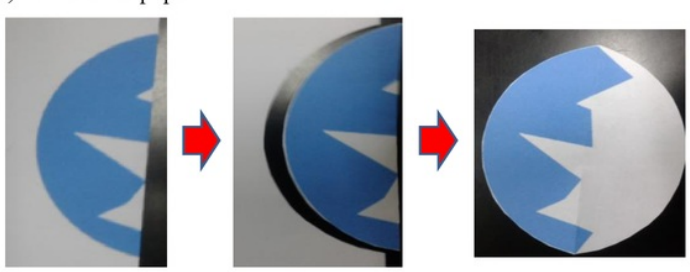
3.1 *Paper-based exploratory task: Unsystematic exploration*

The paper-and-pencil Broken Plate Task was given to 32 student teachers. As an addition to the task, the students received a sheet of paper containing a large figure of the broken plate, blank papers, scissors, compass, and ruler. The student teachers worked in groups of four or five. Every group discussed the mathematical meaning of 'reconstructing the broken plate into its original size and form'. All groups understood that what they had to do was constructing a complete circle which is coincident with the broken plate. At this stage none of the students mentioned the center of circle. They did not notice that to construct the circle they needed to determine the center of the circle. It indicates that the student teachers mainly focused on the shape of a circle and did not really consider the mathematical properties and construction of a circle.

The indication that the student teachers did not really consider the mathematical properties and construction of a circle was confirmed during the next exploratory activity in which they constructed the circle. In the early exploration they did not use mathematical strategies regarding the construction of circle. Instead, they use unsystematic strategies. Here are examples of the student teachers' strategies:

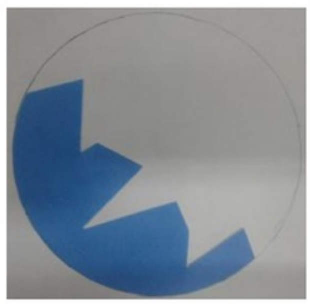
Fold-cut

(1) Fold the paper along the imaginary chord connecting the edges of the arc of the broken plate. In this case the imaginary chord becomes the line of symmetry
(2) Cut the paper along the arc of the broken plate
(3) Unfold the paper



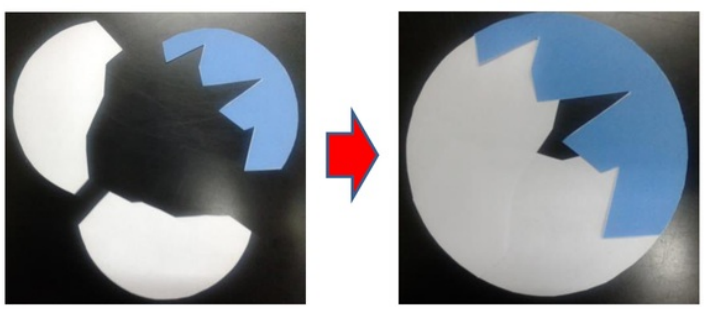
Cut -rotate

(1) Cut the figure of the broken plate
(2) Place it on a paper and trace the arc of the broken plate on the paper
(3) Rotate the broken plate such a way so that a part of its arc coincides with the arc on paper
(4) Trace again the arc of the broken plate on the paper
(5) Repeat step (3) and step (4) until a full circle is constructed



Cut-duplicate-cut-tessellate

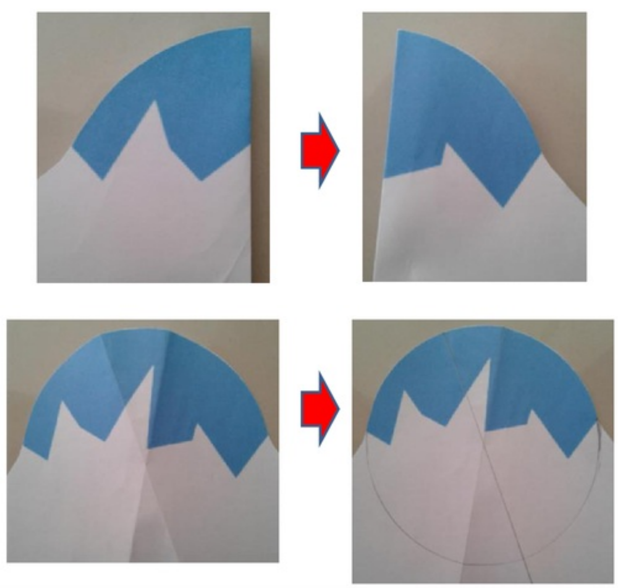
(1) Cut the figure of the broken plate
(2) Make duplicates of the broken plate and cut them
(3) Tessellate the arc of the broken plate and its duplicates in such away to construct a circle



A group of student teachers already included mathematical exploration activity. Although these still used cutting strategy, they already considered mathematical ideas. After half cutting the broken plate, these students folded the plate symmetrically until a line of symmetry was formed. They symmetrically folded the broken plate on other part to get another line of symmetry. They argued that both lines of symmetry must include diameters of the required circle and these diameters intersected at the centre of the circle. At the end, these student teachers constructed a circle by using a compass.

Half-cut – double fold – draw a circle

(1) Cut the broken plate
(2) Fold it symmetrically to get a line of symmetry
(3) Fold it symmetrically to get another line of symmetry
(4) Construct a circle with the intersection of the lines of symmetry as the center

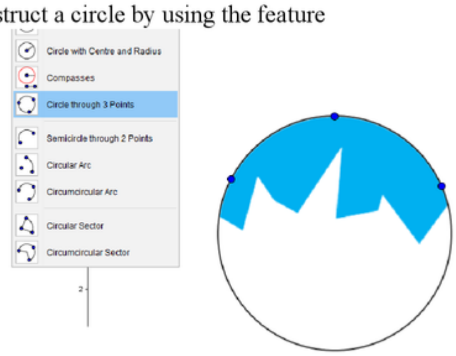


3.2 *GeoGebra-based exploratory task: A tendency toward feature exploration*

Similar to their counterparts who worked on paper-based exploratory task, 19 student teachers who worked on GeoGebra-based exploratory tasks also understood that, mathematically, the task was about constructing a complete circle which was coincident with the broken plate. Between these two groups of participants, the exploratory activity in constructing a circle was rather different. Unlike paper-based task which could lead to non-mathematical exploration, GeoGebra-based exploratory task was more potential toward mathematical exploration. It is reasonable because GeoGebra is already limited to mathematical environment and the features of GeoGebra connect to mathematics concepts. However, from the exploratory activity it seems that these student teachers only focused on the features of GeoGebra and did not really consider mathematics concepts. For example, students who used feature 'Circle through three points' could not give mathematical explanation how a circle can be constructed from three given points. The following are examples of student teachers' feature exploration in constructing a circle.

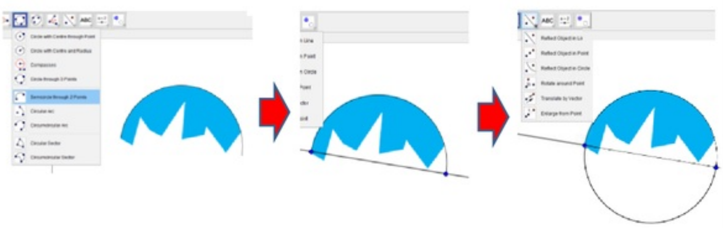
Circle through three points

(1) Choose the feature 'circle through three points'
(2) Put three points along the arc of the broken plate
(3) Construct a circle by using the feature



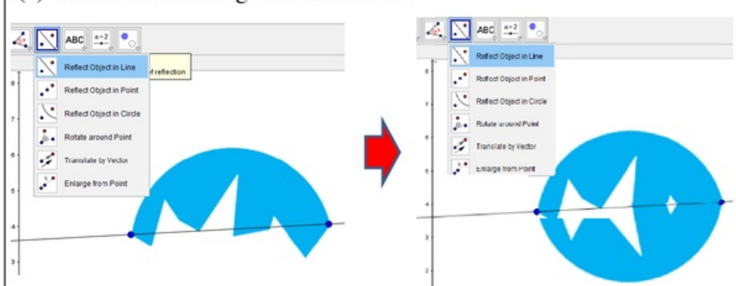
Semi circle – reflect

(1) Choose the feature 'Semicircle through 2 points'
(2) While the feature is activated, put a point on one edge of the arc of the broken plate and construct a semicircle in such a way it coincides the arc of the broken plate
(3) Draw a line connecting the two edges of the semicircle
(4) Choose the feature 'Reflect object in a line'
(5) Click the semicircle and then the line until a circle is constructed



Reflect

(1) Draw a line connecting the two edges of the arc
(2) Choose the feature 'Reflect object in a line'
(3) Click the broken plate and then the line
(4) The constructed figure is not a circle



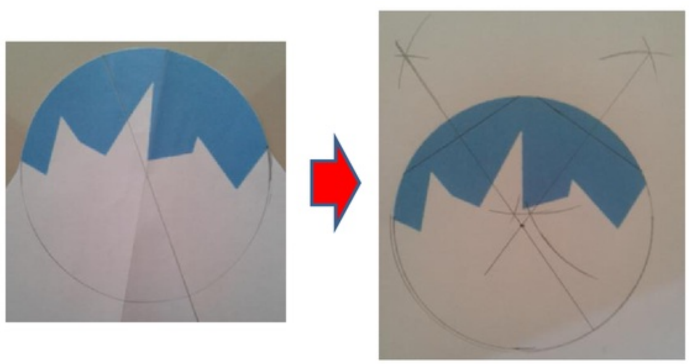
4 Teachers' Scaffolding: From unsystematic exploration and feature exploration to mathematical exploration

The classroom activities show that both exploratory tasks have prompted student teachers to explore strategies, which later could develop students' creativity. However, the early exploratory activity was unsystematic and feature-oriented. Therefore, teacher plays a crucial role to bring this unsystematic and feature exploration toward mathematical exploration. According to Blum (2011), teaching tasks which require mathematical modeling should emphasize on guiding students to construct knowledge or identify mathematics concepts actively and independently by using their prior knowledge and experiences. A key aspect for such teaching is keeping a balance between providing guidance and fostering students' independence. For this purpose, teacher could use flexible interventions and metacognitive prompts to elicit students to reflect on their own understanding of the problem and on how they selected the mathematical procedures to solve the problem. Regarding the use of exploratory tasks in mathematics classrooms, teacher's scaffold could be in the form of questioning. Proper questioning can be used: to focus thinking on mathematics concepts; to help students extend their thinking from concrete and factual knowledge to analytical and evaluative aspect; to help students see connections between different mathematics concepts or between mathematics and real-world contexts (Swan & Pead, 2008). This chapter provides two types of questioning that were used to scaffold student teachers in dealing with exploratory task, i.e. "can you" and "what if".

4.1 *"Can you" question*

The first question that was used to guide student teachers to do mathematical exploration was "**can you** find other strategy?" It was observed that this question directed students to explore strategies. However, their new strategies were still at the same level, either unsystematic or feature-oriented. For example, students who used 'circle through 3 points' came up with 'semi circle and reflect' as a new strategy without giving any explanation related to circle's mathematical properties. It seems that the words 'other strategies' were not specific enough to direct student teachers to think of the mathematical properties and construction of a circle. Considering this response, a more specific "can you" question was posed, e.g. "Can you do it without cutting the broken plate?" which was posed to groups of student teachers who worked with paper-based tasks. By limiting students' strategy, which in this case was cutting, directed student teachers to explore other tools and strategies. A group of students who previously came up with the strategy 'half-cut – double fold – draw a circle' discussed how they could construct lines of symmetry without cutting and folding paper. These students finally noticed that the lines of symmetry could be constructed by drawing the medians of chords as shown in the following figures. (Note: constructing apothems also observed in the groups of student teachers who worked with GeoGebra-based task).

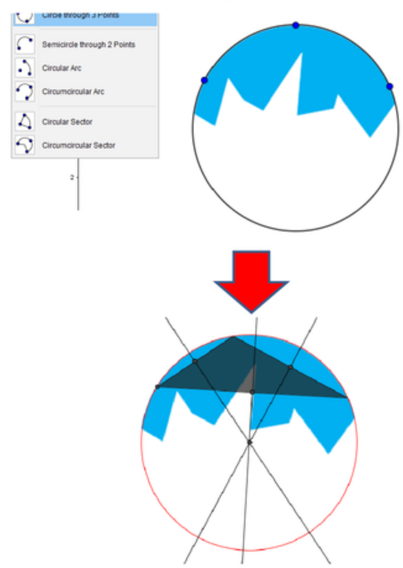
"Can you do it without cutting the broken plate?"



4.2 *"What if ..." strategy*

According to Kaur (2012), a "What if" task which included a modified given information could direct students to re-examine the task and see the impact of the changes of the task on the solution process. Taking a similar perspective, a “what if” questioning was posed to direct students to mathematical exploration. To some student teachers who worked with GeoGebra-based exploratory task, the question “what would you do if we use compass instead of GeoGebra?” The modified information in this questioning is GeoGebra, which is replaced by compass (note: the compass was imaginary because student teachers were still working with GeoGebra). After group discussion, student teachers who previously used ‘Circle through 3 points’ strategy noticed that three points on the arc of the broken circle meant a triangle. As their new strategy, these students constructed a triangle whose vertex lie on the broken plate and then constructed a circumscribed circle of this triangle. (Note: this circumscribed circle was also emerged in the groups of students who worked with paper-based task after teacher scaffold with questioning strategy).

“What would you do if we use compass instead of GeoGebra?”



5 Concluding Remark

This chapter provides an example of exploratory task is modified from a textbook task. A textbook task often provides only information required to solve it and also gives a clear indication about the required mathematics concepts or procedures. Such task can be modified into an exploratory task by reducing some information and making it ill-structured. Furthermore, an exploratory task can be presented as a paper-based task or a computer-based task. An exploratory task could lead students to unsystematic and non-mathematical exploration. Therefore, the effectiveness of an exploratory task to empower mathematics learners also depends on teachers' scaffold during students' exploratory activity. Questioning is scaffolding that can guide students toward mathematical exploration while keeping students' active and independent work.a specific "Can you" questioning and "What if …" questioning were found to be effective to bring students from unsystematic and feature-oriented exploration to mathematical exploration.

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